Exercise 45

A warm can of soda is placed in a cold refrigerator. Sketch the graph of the temperature of the soda as a function of time. Is the initial rate of change of temperature greater or less than the rate of change after an hour?

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Solution

According to Newton's law of cooling, the rate that the can's temperature changes is proportional to the temperature difference of the can and the surrounding environment.

$$\frac{dT}{dt} \propto (T - T_a)$$

Here T_a is the ambient temperature. Change this to an equation by introducing a proportionality constant h.

$$\frac{dT}{dt} = -h(T - T_a)$$

The negative sign is included so that when the can is warmer dT/dt is negative and vice-versa. Solving this equation yields

$$T(t) = T_a + (T_i - T_a)e^{-ht},$$

where T_i is the can's initial temperature. If the can's temperature is 100 and the ambient temperature is 50, for example, then the function reduces to

$$T(t) = 50 + 50e^{-ht}.$$

A graph of this function is shown below with $h = 1$ to illustrate the behavior.
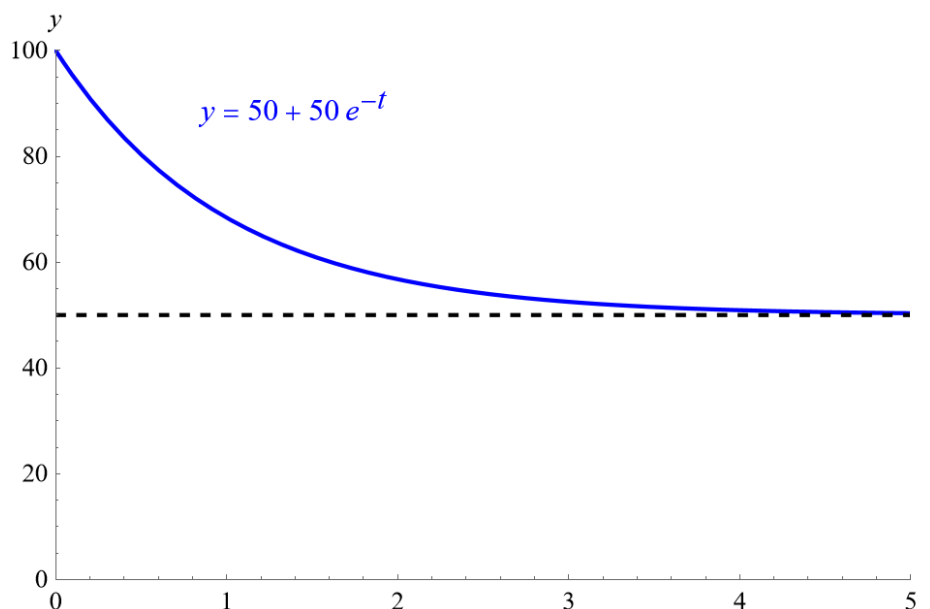
The initial rate of change of temperature is less than the rate of change after an hour.

$$\frac{dT}{dt}(0) < \frac{dT}{dt}(1)$$